

# Vacuum Instability in Chern-Simons Gravity

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We explore perturbations about a Friedmann-Robertson-Walker background in Chern-Simons gravity. At large momenta one of the two circularly polarized tensor modes becomes ghostlike. We argue that nevertheless the theory does not exhibit classical runaway solutions, except possibly in the relativistic nonlinear regime. However, the ghost modes cause the vacuum state to be quantum mechanically unstable, with a decay rate that is naively infinite. The decay rate can be made finite only if one interprets the theory as an effective quantum field theory valid up to some momentum cutoff  $\Lambda$ , which violates Lorentz invariance. By demanding that the energy density in photons created by vacuum decay over the lifetime of the Universe not violate observational bounds, we derive strong constraints on the two dimensional parameter space of the theory, consisting of the cutoff  $\Lambda$  and the Chern-Simons mass.

## I. INTRODUCTION AND SUMMARY

General relativity has held up well to various tests over the years from experiments and astronomical observations [1], and is considered a pillar of standard cosmology. However, it is interesting to consider modifications to the theory, particularly in light of the observed acceleration of the Universe [2]. One useful approach is to think of Einstein gravity as an effective field theory, and consider higher order corrections to the Einstein-Hilbert action, either involving the metric alone or involving an additional posited scalar field. The goal then becomes to calculate the corrections to general relativity arising from these higher order terms, and using experiments to set bounds on the couplings parameterizing them.

One such extension to general relativity is Chern-Simons gravity [3, 4], where one assumes the existence of a canonical scalar field, coupled to gravity through a parity violating term. The theory is described by the action

$$S = S_{EH} + S_{CS} + S_{\vartheta} + S_{\text{mat}}, \quad (1.1)$$

where the various terms are respectively the Einstein-Hilbert term

$$S_{EH} = \frac{1}{2} m_{\text{p}}^2 \int d^4x \sqrt{-g} R, \quad (1.2a)$$

the Chern-Simons term

$$S_{CS} = \frac{1}{4} \alpha \int d^4x \sqrt{-g} \vartheta {}^*RR, \quad (1.2b)$$

the scalar term

$$S_{\vartheta} = -\frac{1}{2} \int d^4x \sqrt{-g} [\nabla_a \vartheta \nabla^a \vartheta + 2V(\vartheta)], \quad (1.2c)$$

and  $S_{\text{mat}}$  describes any other matter present. In these expressions  $m_{\text{p}}^2 = (8\pi G)^{-1}$  is the square of the reduced Planck mass,  $g$  is the determinant of the metric,  $R$  is the Ricci scalar and  $\alpha$  a coupling constant with dimensions

of inverse mass. Also  $V(\vartheta)$  is an arbitrary potential and the Pontryagin density is defined as

$${}^*RR = \frac{1}{2} \epsilon^{cdef} R^a_{\text{bef}} R^b_{\text{acd}}, \quad (1.3)$$

where  $\epsilon^{cdef}$  is the four dimensional Levi-Civita tensor. For purposes of the present discussion, we will assume that matter is minimally coupled to the metric.

Consider now the dynamics of Chern-Simons gravity in perturbation theory about a Friedman-Robertson-Walker (FRW) cosmological background. Now in the limit  $\vartheta = \text{constant}$ , the Chern-Simons term (1.2b) reduces to a surface term and we recover Einstein's equations for the space-time dynamics. Therefore, in the effective field theory that describes the perturbations, the operators that arise from the Chern-Simons term must be suppressed by a mass scale that is related to the derivative of the background scalar field. This mass scale is called the Chern-Simons mass  $m_{\text{cs}}$ , and is defined by [5]

$$m_{\text{cs}} \equiv \frac{m_{\text{p}}^2}{\alpha \dot{\vartheta}}, \quad (1.4)$$

where the dot denotes a derivative with respect to time. General relativity is recovered in the limit  $m_{\text{cs}} \rightarrow \infty$ . Because it is the Chern-Simons mass that enters into equations describing observables, we choose to constrain it, rather than the more fundamental coupling  $\alpha$  that appears in the action (1.2b).

Past work constraining Chern-Simons gravity has utilized Solar System tests of general relativity. Measurement of Lense-Thirring precession by LAGEOS give the bound [5]

$$m_{\text{cs}} \gtrsim 10^{-13} \text{ eV}. \quad (1.5)$$

A bound  $10^{11}$  times stronger has been claimed from binary pulsar studies [6], but the validity of this result has been questioned [7], and a corrected bound from binary pulsars is [7]

$$m_{\text{cs}} \gtrsim 5 \times 10^{-10} \text{ eV}. \quad (1.6)$$

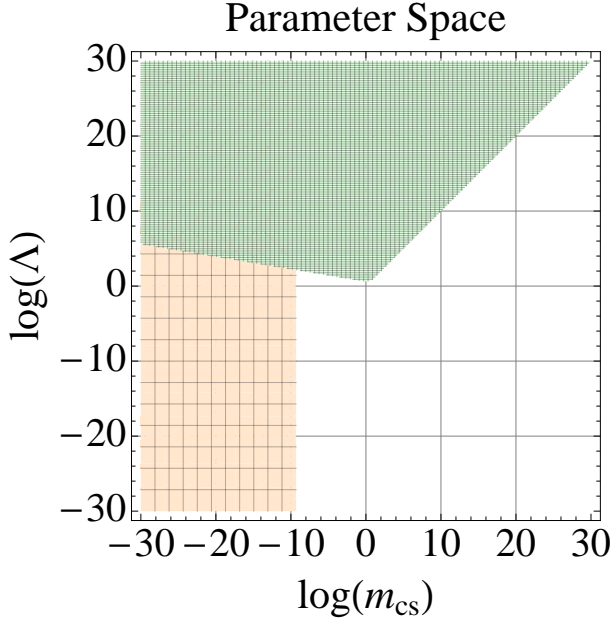


FIG. 1: Effective field theory parameter space consisting of the cutoff scale  $\Lambda$  and the Chern-Simons mass  $m_{cs}$ , in eV. The lightly shaded region is excluded from binary pulsar observations [7], and the darkly shaded region is excluded by consideration of vacuum decay.

In this paper we study the vacuum stability of Friedmann-Robertson-Walker (FRW) solutions in Chern-Simons gravity as a function of the Chern-Simons mass parameter. In Section II we consider tensor perturbations to the FRW metric. We show that for spatial momenta above the Chern-Simons mass scale, one of the two polarization modes is ghostlike and can decay to radiation. Requiring that the radiation produced over the lifetime of the Universe not exceed observational bounds will allow us to constraint the parameters of the theory.

In Section IV we argue that the theory does not exhibit classical runaway solutions, despite the existence of ghostlike modes, except possibly in the relativistic non-linear regime.

## II. THE EXISTENCE OF GHOST GRAVITON MODES

It is easily verified that an FRW metric, with scale factor and matter sources obeying the usual Friedmann equations, is a solution of the equations of motion of the theory (1.1) [8], as the FRW symmetries lead to a vanishing Pontryagin density. Consider now linear perturbations about such a solution. As in general relativity, the symmetries of the background solution guarantee that perturbations can be decomposed into scalar, vector and tensor modes.

In this paper we will focus on tensor perturbation modes, for which the perturbation to the scalar field van-

ishes and the metric takes the form

$$ds^2 = a^2(\eta)[-d\eta^2 + (\delta_{ij} + h_{ij})d\chi^i d\chi^j]. \quad (2.1)$$

Here  $a(\eta)$  is the scale factor,  $\eta$  is conformal time and  $\chi^i$  are comoving coordinates. We adopt the transverse and traceless gauge conditions,  $h^i_i = 0$ ,  $\partial_i h^{ij} = 0$ . Expanding the terms in the action (1.1) to quadratic order yields [9]

$$S = \frac{1}{8} \int d\eta d^3\chi \left[ m_p^2 a^2(\eta) (h^i_{j,\eta} h^j_{i,\eta} - h^i_{j,k} h^j_{i,k}) - \alpha \vartheta_{,\eta} \epsilon^{ijk} (h^q_{i,\eta} h_{kq,j\eta} - h^q_{i,r} h_{kq,rj}) \right] + O(h^3), \quad (2.2)$$

where  $\epsilon^{ijk}$  is the Levi-Cevita symbol. We rewrite this action in terms of the Fourier transform of the metric perturbation, which is defined by

$$h_{ij}(\eta, \mathbf{x}) = \int d^3k \tilde{h}_{ij}(\eta, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (2.3)$$

where  $\mathbf{k}$  is the comoving wavevector. We define the left and right circular polarization modes  $\tilde{h}_{A\mathbf{k}}(\eta)$  by

$$\tilde{h}_{ij}(\eta, \mathbf{k}) = \sum_{A=L,R} \tilde{h}_{A\mathbf{k}}(\eta) e^A_{ij}(\mathbf{n}). \quad (2.4)$$

where  $\mathbf{n} = \mathbf{k}/k$  is a unit vector in the direction of propagation,  $k \equiv |\mathbf{k}|$ , and the polarization tensors  $e^A_{ij}(\mathbf{n})$  satisfy the conditions

$$e^A_{ij} (e^B_{ij})^* = 2\delta^{AB}, \quad (2.5a)$$

$$n_i \epsilon^{ijk} e^A_{kl} = i\lambda_A (e^j_l)^A, \quad (2.5b)$$

with  $\lambda_R = +1$  and  $\lambda_L = -1$ . The action can now be written as

$$S = \frac{m_p^2}{4} \int d\eta d^3k \sum_{A=L,R} a^2(\eta) \left[ 1 + \frac{\lambda_A k}{a^2(\eta)} \frac{\alpha \vartheta_{,\eta}}{m_p^2} \right] \left[ |\tilde{h}_{A\mathbf{k},\eta}|^2 - k^2 |\tilde{h}_{A\mathbf{k}}|^2 \right]. \quad (2.6)$$

This can be recast using the Chern-Simons mass scale (1.4) as

$$S = \frac{m_p^2}{4} \int d\eta d^3k \sum_{A=L,R} a^2(\eta) \left[ 1 + \lambda_A \frac{k_{\text{phy}}}{m_{cs}} \right] \left[ |\tilde{h}_{A\mathbf{k},\eta}|^2 - k^2 |\tilde{h}_{A\mathbf{k}}|^2 \right], \quad (2.7)$$

where  $k_{\text{phy}} = k/a$  is the physical wavenumber. The action (2.7) is the usual action for tensor perturbations in FRW, except for the momentum dependent correction factor  $(1 + \lambda_A k_{\text{phy}}/m_{cs})$ . This factor becomes negative

for the left handed polarization modes when  $k_{\text{phy}} \geq m_{\text{cs}}$ , giving rise to a kinetic term with the wrong sign, i.e. a ghost mode.

The action (2.7) can be simplified by changing the normalization of the graviton modes to attain canonical normalization. We define

$$\varepsilon_A(k) = \text{sgn} \left( 1 + \frac{\lambda_A k_{\text{phys}}}{m_{\text{cs}}} \right), \quad (2.8)$$

which is +1 for normal modes and -1 for ghost modes. We define the mass scale

$$m_* = m_{\text{p}} \sqrt{\left| 1 + \frac{\lambda_A k_{\text{phys}}}{m_{\text{cs}}} \right|}, \quad (2.9)$$

and the canonically normalized graviton field modes

$$\tilde{h}_{A\mathbf{k}}^{\text{can}} = m_* \tilde{h}_{A\mathbf{k}}. \quad (2.10)$$

The action (2.7) can now be written as

$$S = \frac{1}{4} \int d\eta \, d^3k \sum_{A=L,R} a^2(\eta) \varepsilon_A(k) \left[ \left| \tilde{h}_{A\mathbf{k},\eta}^{\text{can}} - (\ln m_*)_{,\eta} \tilde{h}_{A\mathbf{k}}^{\text{can}} \right|^2 - k^2 |\tilde{h}_{A\mathbf{k}}^{\text{can}}|^2 \right]. \quad (2.11)$$

Note that from a classical point of view the existence of ghost modes does not necessarily imply any inconsistency of the theory. The Hamiltonian of the theory may or may not be unbounded below; addressing this question would require a nonlinear analysis beyond the scope of this paper. Also it is not known at present whether the theory possesses a well posed initial value formulation: the sign flip at  $k_{\text{phys}} = m_{\text{cs}}$  may be a hint that it does not. However, the ghost modes are a significant problem when quantum mechanical effects are taken into account, as we discuss in the next section.

One might also expect that the ghost graviton modes would be associated with fluxes of negative energy. Rather surprisingly, this is not so; a computation of the effective stress energy tensor for gravitational waves at future null infinity shows that all the graviton modes carry positive energy [10].

### III. CONSTRAINTS FROM VACUUM DECAY

We now specialize to perturbation modes today which are deep inside the horizon, that is,  $k \gg H_0$ , where  $H_0 = a_{,\eta}/a^2$  is the Hubble parameter. We also assume that  $m_{\text{cs}} \gg H_0$ . In this limit, we can neglect in the action (2.11) the time dependence of the prefactor  $a(\eta)^2$ , and also the term proportional to the time derivative of  $m_*$ . The result is just the standard action for graviton modes in Minkowski spacetime, except for the sign flip for the ghost modes<sup>1</sup>.

Because of this sign flip, decay of the vacuum of the theory is kinematically allowed. The vacuum decay rate per unit volume  $\Gamma$  is naively infinite, because of the infinite phase space available for the decay products that arises from Lorentz invariance<sup>2</sup>. Rather than ruling out the theory outright because of the presence of ghost modes, we adopt the viewpoint that the action (2.7) defines an effective quantum field theory with some effective cutoff  $\Lambda$  on the physical wavenumber  $k_{\text{phys}}$  in cosmological rest frame. Thus, our cutoff explicitly violates Lorentz invariance; such a violation is inevitable if one wants to obtain a finite vacuum decay rate. Our viewpoint and treatment follow similar analyses of scalar ghost fields in cosmological models with equation of state parameter  $w$  that satisfies  $w < -1$  [11, 12]. As in those analyses, we will find that stringent constraints on the parameters of the theory can be obtained by demanding that the total number of photons produced from vacuum decay over the lifetime of the Universe not be in conflict with observations.

Our theory is now parameterized by two parameters, the cutoff  $\Lambda$  and the Chern-Simons mass  $m_{\text{cs}}$ . Now since the ghost modes arise only for momenta  $k_{\text{phys}}$  satisfying  $k_{\text{phys}} > m_{\text{cs}}$ , there will be no ghost modes in the region  $m_{\text{cs}} > \Lambda$  of parameter space. Therefore our arguments about vacuum decay do not constrain that region of parameter space; we focus from now on on the complementary region  $m_{\text{cs}} < \Lambda$  (see Fig. 1). We note that many papers on Chern-Simons gravity implicitly work in this regime  $m_{\text{cs}} < \Lambda$ , since they involve effects arising on scales  $k_{\text{phys}} \sim m_{\text{cs}}$ , assumed to be within the domain of validity of the theory.

If the vacuum perturbatively decays to stable particles, we can estimate the number of particles produced over the lifetime of the Universe from such a process. Given observational bounds on the energy density in this particle species, we can then constrain the decay rate. The strongest bounds will therefore come from production of particles whose energy density has been reliably measured, and which can be produced at a low order in perturbation theory. We argue that the photon is the best such candidate.

We next make an order of magnitude estimate of the decay rate  $\Gamma$  per unit volume to photons in the regime  $m_{\text{cs}} \ll \Lambda$ . The action for electromagnetism contains an interaction of the form

$$S_{\text{int}} \sim \int d^4x h (\partial A)^2 \sim \frac{1}{m_*} \int d^4x h^{\text{can}} (\partial A)^2, \quad (3.1)$$

where  $A^a$  is the 4-vector potential,  $h$  is the metric per-

the standard form when written in terms of  $\tilde{h}_{A\mathbf{k}}$ , but acquires correction factors of  $m_{\text{p}}/m_*$  when written in terms of  $\tilde{h}_{A\mathbf{k}}^{\text{can}}$ .

<sup>2</sup> The full theory of the perturbations about FRW, including coupling to matter, does violate Lorentz invariance because of the  $k_{\text{phys}}$  dependence of the factor  $1 + \lambda_A k_{\text{phys}}/m_{\text{cs}}$ . However this violation does not affect the accessible volume of phase space.

<sup>1</sup> We have omitted the action describing the interaction of the graviton field modes with matter fields. This interaction takes

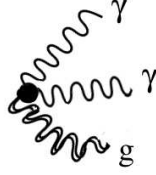


FIG. 2: Spontaneous production of photons and ghost gravitons from vacuum

turbation, and  $h^{\text{can}}$  is the canonically normalized version. Because the graviton is ghostlike, the process

$$0 \rightarrow g\gamma\gamma \quad (3.2)$$

is kinematically allowed, where  $g$  is a left polarized graviton and  $\gamma$  is a photon (Fig. 2). Hence graviton ghosts can decay to photons at first order in perturbation theory.

Next, the coefficient  $1/m_*$  in the interaction (3.1) depends on the wavenumber  $k$ . However, for the purposes of our order of magnitude estimate, it will be sufficient to evaluate this coefficient at  $k_{\text{phys}} \sim \Lambda$ , since most of the decays will be at  $k_{\text{phys}} \sim \Lambda$ . Therefore we can treat  $m_*$  as a constant,

$$m_* \sim m_p \sqrt{\frac{\Lambda}{m_{\text{cs}}}}, \quad (3.3)$$

where we have used  $m_{\text{cs}} \ll \Lambda$  and specialized to ghost modes. The decay rate  $\Gamma$  per unit time per unit volume must be proportional to the square of the coefficient of the operator, so  $\Gamma \propto 1/m_*^2$ . The constant of proportionality in this relation must be some function of  $\Lambda$ , and from dimensional analysis it now follows that

$$\Gamma_{0 \rightarrow g\gamma\gamma} \sim \frac{\Lambda^6}{m_*^2} \sim \frac{m_{\text{cs}} \Lambda^5}{m_p^2}, \quad (3.4)$$

since  $\Gamma$  has dimension (mass)<sup>4</sup> in units with  $\hbar = c = 1$ .

Consider now the production of photons by this mechanism over the lifetime of the Universe. We make the idealization that at any epoch, all the photons produced have an energy of exactly  $\Lambda$ . Then the evolution of the number  $n(k, t)$  of photons per unit logarithmic wavenumber  $k$  per proper volume is given by

$$\frac{1}{a^3} \frac{d}{dt} (a^3 n) = \Gamma \Lambda \delta(k/a - \Lambda), \quad (3.5)$$

where  $a$  is the scale factor with  $a = 1$  today. Solving this equation gives for the spectrum today

$$n(k) = \Theta(\Lambda - k) \frac{\Gamma a_*^3}{H_*}, \quad (3.6)$$

where  $\Theta$  is the step function,  $H = \dot{a}/a$  is the Hubble parameter, and  $a_*(k)$  and  $H_*(k)$  are the values of  $a$  and  $H$  at  $k/a = \Lambda$ . The result (3.6) is easy to understand: the

factor of  $1/H_*$  is the length of time during which photons of present-day energy  $\sim k$  are produced, and the factor of  $a_*^3$  is the volume expansion factor since then. For a  $\Lambda$ CDM cosmology with  $H^2 = H_0^2(\Omega_M/a^3 + 1 - \Omega_M)$  the spectrum becomes

$$n(k) = \Theta(\Lambda - k) \frac{\Gamma}{H_0} \left[ \Omega_M \left( \frac{\Lambda}{k} \right)^9 + (1 - \Omega_M) \left( \frac{\Lambda}{k} \right)^6 \right]^{-1/2}. \quad (3.7)$$

It follows that the energy density per unit logarithmic wavenumber  $dE/d^3x d \ln k \sim k n(k)$  is peaked at  $k \sim \Lambda$  with a peak value of

$$\frac{dE}{d^3x d \ln k} \sim \frac{\Gamma \Lambda}{H_0} \sim \frac{m_{\text{cs}} \Lambda^6}{m_p^2 H_0}. \quad (3.8)$$

We now compare the prediction (3.8) with observational data. Observations in various wavelength bands from radio to gamma rays gives the upper bound<sup>3</sup> on the background radiation spectrum

$$\frac{dE}{d^3x d \ln k} \lesssim (2.5 \times 10^{-5} \text{ eV})^4 \quad (3.9)$$

for  $10^{-10} \text{ eV} \lesssim k \lesssim 10 \text{ GeV}$  [13, 14]. Combining this with the prediction (3.8) yields the constraint

$$m_{\text{cs}} \Lambda^6 \lesssim (3.3 \text{ eV})^7, \quad (3.10)$$

for  $10^{-10} \text{ eV} \lesssim \Lambda \lesssim 10 \text{ GeV}$ , where we have used  $m_p = 2.4 \times 10^{27} \text{ eV}$  and  $H_0 = 1.5 \times 10^{-33} \text{ eV}$ . Figure 1 shows the constraints (1.6) and (3.10) on the  $(m_{\text{cs}}, \Lambda)$  parameter space. It can be seen that the combined constraints rule out a large part of the region in parameter space with  $\Lambda > m_{\text{cs}}$  in which interesting Chern-Simons phenomenology occurs, except for the window  $10^{-10} \text{ eV} \lesssim m_{\text{cs}} \lesssim 3 \text{ eV}$ .

One can also compute the constraints obtained from vacuum decay to gravitons, in which one ghost graviton and two normal gravitons are produced. The interaction Lagrangian is

$$\mathcal{L}_{\text{int}} \sim m_p^2 h (\partial h)^2 \sim \frac{m_p^2}{m_*^3} h_{\text{can}} (\partial h_{\text{can}})^2. \quad (3.11)$$

The analysis now proceeds as before, with the resulting energy density in gravitons given by

$$\Omega_{\text{gw}} \sim \frac{1}{m_p^2 H_0^2} \frac{dE}{d^3x d \ln k} \sim \frac{\Lambda^4 m_{\text{cs}}^3}{m_p^4 H_0^3} \quad (3.12)$$

at  $k \sim \Lambda$ . Now observations of the cosmic microwave background give an integrated upper bound  $\int d \ln k \Omega_{\text{gw}} \lesssim 10^{-5}$  for  $k \gtrsim 10^{-30} \text{ eV}$  [15]. However this

<sup>3</sup> This upper bound is exceeded by about two orders of magnitude by the CMB, but the CMB is known to be thermal to about 1 part in  $10^5$ , so the bound effectively holds.

bound applies only to the gravitons produced before recombination, whose energy density will be smaller than the estimate (3.12) by a factor of the ratio of the age of the Universe at recombination to the age of the Universe today. This factor is approximately  $z_{\text{rec}}^{-3/2}$ , where  $z_{\text{rec}} \sim 1000$  is the redshift of recombination. We therefore obtain the constraint

$$m_{\text{cs}}^3 \Lambda^4 \lesssim z_{\text{rec}}^{3/2} m_{\text{p}}^4 H_0^3 (10^{-5}) \sim (32 \text{ eV})^7 \quad (3.13)$$

for  $\Lambda \gtrsim 10^{-30} \text{ eV}$ , which is slightly weaker than the constraint (3.10) from photons.

#### IV. CLASSICAL RUNAWAY SOLUTIONS

We next turn to a different question, the possible existence of classical runaway solutions that can occur in a theory whose Hamiltonian is unbounded below. We will argue that such solutions do not arise in Chern-Simons gravity except possibly in the nonlinear relativistic regime  $h \sim 1$  where our analysis is invalid anyway.

Generically, a Hamiltonian which is unbounded from below will only exhibit runaway solutions in the regime where the interaction terms in the Lagrangian are comparable to the negative kinetic energy terms [11]. We use this criterion to estimate the required occupation number of a ghost graviton mode for such a runaway behavior to develop in our system.

No runaway solutions will plague the effective field theory (2.7) because the ghost and non-ghost fields are decoupled to linear order. However, interaction terms will appear if we expand the action (1.1) to higher than quadratic order. Expanding up to quartic order we expect to find interaction terms of the form

$$S_3 \sim \int d^4x m_{\text{p}}^2 k^2 h_L^2 h_R + \dots, \quad (4.1a)$$

$$S_4 \sim \int d^4x m_{\text{p}}^2 k^2 h_L^2 h_R^2 + \dots \quad (4.1b)$$

Consider a single localized wave packet mode with characteristic size  $\lambda$ . We would like to estimate the number of quanta  $N$  for an instability to form. The energy from the kinetic term is

$$E_K = \frac{1}{2} \int d^3x m_{\text{p}}^2 \dot{h}^2 \sim \lambda m_{\text{p}}^2 h^2 \sim \frac{N}{\lambda}. \quad (4.2)$$

Therefore the field fluctuation is approximately

$$h \sim \frac{N^{1/2}}{\lambda m_{\text{p}}}. \quad (4.3)$$

Each of the interaction terms will therefore contribute terms to the energy on the order of

$$E_3 \sim \int d^3x m_{\text{p}}^2 k^2 h^3 \sim \frac{N^{3/2}}{\lambda^2 m_{\text{p}}}, \quad (4.4a)$$

$$E_4 \sim \int d^3x m_{\text{p}}^2 k^2 h^4 \sim \frac{N^2}{\lambda^3 m_{\text{p}}^2}. \quad (4.4b)$$

Requiring that the kinetic energy be less than the interaction energy

$$E_3 \gtrsim E_K, \quad E_4 \gtrsim E_K, \quad (4.5)$$

and taking the wavelength  $\lambda \sim m_{\text{cs}}^{-1}$  yields

$$N \geq \left( \frac{m_{\text{p}}}{m_{\text{cs}}} \right)^2 \gg 1. \quad (4.6)$$

We deduce from (4.3) that in this regime

$$h \gtrsim 1. \quad (4.7)$$

We conclude the ghost modes generated by perturbations to FRW do not exhibit runaway solutions within the domain of validity of our analysis.

#### V. OUTLOOK

We have shown that if the effective field theory cutoff  $\Lambda$  is above the Chern-Simons mass scale  $m_{\text{cs}}$ , the vacuum of Chern-Simons gravity is unstable and can decay to photons. Our estimate of the photon production rate was used to set constraints on the parameter space. One might imagine improving this bound by being less cavalier in the estimation of the decay rate and number density. However, because the production rate goes like a high power of the cutoff scale, orders of magnitude difference in the number density yield very modest improvements in final constraints on  $\Lambda$  and  $m_{\text{cs}}$ .

Whether the full nonlinear theory has a Hamiltonian which is unbounded below is an interesting open question.

Finally, we note that it is possible that the decay of the vacuum could be a lot faster than indicated by our perturbative calculations, if it is mediated by a non-perturbative process. One might imagine a tunneling process similar to the decay of false vacua in scalar field theories via bubble nucleation. In particular, a homogeneous unstable region with small Chern-Simons might be expected to produce a stable endstate consisting of a homogeneous region with a Chern-Simons mass larger than the cutoff. However any such transition could not proceed via a spherically symmetric process, since the Chern-Simons term does not contribute to the dynamics in spherical symmetry.

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